

Einstein-singleton theory and its power spectra in de Sitter inflation

Yun Soo Myung^{a*}, Taeyoon Moon^{a†}, and Young-Jai Park^{b‡}

^aInstitute of Basic Sciences and Department of Computer Simulation, Inje University
Gimhae 621-749, Korea

^bDepartment of Physics, Sogang University, Seoul 121-742, Korea

Abstract

We study the Einstein-singleton theory during de Sitter inflation since it provides a way of degenerate fourth-order scalar theory. We obtain an exact solution expressed in terms of the exponential-integral function by solving the degenerate fourth-order scalar equation in de Sitter spacetime. Furthermore, we find that its power spectrum blows negatively up in the superhorizon limit, while it is negatively scale-invariant in the subhorizon limit. This suggests that the Einstein-singleton theory contains the ghost-instability and thus, it is not suitable for developing a slow-roll inflation model.

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*e-mail address: ysmjung@inje.ac.kr

†e-mail address: tymoon@inje.ac.kr

‡e-mail address: yjpark@sogang.ac.kr

1 Introduction

The single-field inflation is still known to be a promising model for describing the slow-roll (quasi-de Sitter) inflation [1] when one chooses an appropriate potential like the Starobinsky potential which originates from $f(R) = R + R^2$ gravity [2]. This Einstein-scalar theory corresponds to a second-order tensor-scalar theory.

Our next question is to consider an Einstein-(higher-order) scalar theory even though one may worry about a ghost state. For this purpose, it was interesting to compute the power spectrum of a massive singleton (other than inflaton) generated during de Sitter (dS) inflation because its equation belongs to a fourth-order equation. In order to compute the power spectrum, one has to choose the Bunch-Davies vacuum in the subhorizon limit of $z \rightarrow \infty$. In addition, one needs to quantize the singleton canonically as the inflaton did. However, it is hard to obtain a fully exact solution to the fourth-order equation in dS spacetime. Instead, the authors in [3] have investigated the massive singleton to show the dS/LCFT correspondence in the superhorizon limit of $z \rightarrow 0$ as an extension to the dS/CFT correspondence. Recently, two of us have shown that the momentum correlators of LCFT take the same form as the power spectra $\propto k^3$ in the superhorizon limit [4]. This might show that the dS/LCFT correspondence works for obtaining the power spectra in the superhorizon limit. Nevertheless, the limitation of these works is that their computations are valid only in the superhorizon limit because of difficulty in solving a fourth-order differential equation in whole range z .

In this work we obtain an exact solution and compute a complete power spectrum of singleton by solving the degenerate fourth-order scalar equation, which describes a propagation of a massless singleton during dS inflation and by requiring the Pais-Uhlenbeck quantization scheme for a degenerate fourth-order oscillator [5, 6, 7]. It turns out that the singleton power spectrum blows negatively up in the superhorizon limit, while it is negatively scale-invariant in the subhorizon limit. This suggests that the Einstein-singleton theory is not a candidate for a slow-roll inflation because its power spectrum might show ghost-instability.

2 Einstein-singleton theory

We introduce the Einstein-singleton theory where a dipole ghost pair ϕ_1 and ϕ_2 are minimally coupled to Einstein gravity. The starting action is a second-order scalar-tensor theory given by

$$S_{\text{ES}} = S_{\text{E}} + S_{\text{S}} = \int d^4x \sqrt{-g} \left[\left(\frac{R}{2\kappa} - 2\Lambda \right) - \left(\partial_\mu \phi_1 \partial^\mu \phi_2 + \frac{\mu}{2} \phi_1^2 \right) \right], \quad (1)$$

where S_{E} is introduced to feed the dS inflation with $\Lambda > 0$ and S_{S} represents the singleton theory composed of two scalars ϕ_1 and ϕ_2 [8, 9, 10, 11, 12]. Here we have $\kappa = 8\pi G = 1/M_{\text{P}}^2$ with the reduced Planck mass M_{P} and μ is a coupling parameter.

After the metric variation, the Einstein equation is given by

$$G_{\mu\nu} + \kappa\Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \quad (2)$$

with the energy-momentum tensor

$$T_{\mu\nu} = 2\partial_\mu \phi_1 \partial_\nu \phi_2 - g_{\mu\nu} \left(\partial_\mu \phi_1 \partial^\mu \phi_2 + \frac{\mu}{2} \phi_1^2 \right). \quad (3)$$

Importantly, two scalar fields are coupled to be

$$\nabla^2 \phi_1 = 0, \quad \nabla^2 \phi_2 = \mu \phi_1, \quad (4)$$

which lead to a degenerate fourth-order equation

$$\nabla^4 \phi_2 = 0. \quad (5)$$

It can describe a fourth-order scalar theory because S_{S} reduces to the fourth-order scalar theory when eliminating an auxiliary field ϕ_1 as [13]

$$S_{\text{S}}^4 = \frac{1}{2\mu} \int d^4x \sqrt{-g} \nabla^2 \phi_2 \nabla^2 \phi_2, \quad (6)$$

which provides (5) directly. Choosing the vanishing scalars, the solution of dS spacetime comes out as

$$\bar{R} = 4\kappa\Lambda, \quad \bar{\phi}_1 = \bar{\phi}_2 = 0. \quad (7)$$

Explicitly, dS-curvature quantities are given by

$$\bar{R}_{\mu\nu\rho\sigma} = H^2 (\bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma} \bar{g}_{\nu\rho}), \quad \bar{R}_{\mu\nu} = 3H^2 \bar{g}_{\mu\nu} \quad (8)$$

with a Hubble parameter $H = \sqrt{\kappa\Lambda/3}$. We select the dS background explicitly by choosing a conformal time η

$$ds_{\text{dS}}^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = a(\eta)^2 [-d\eta^2 + \delta_{ij} dx^i dx^j], \quad (9)$$

where the conformal and cosmic scale factors are given by

$$a(\eta) = -\frac{1}{H\eta}, \quad a(t) = e^{Ht}. \quad (10)$$

During the dS inflation, $a(\eta)$ goes from small to a very large value like $a_f/a_i \simeq 10^{30}$, which corresponds to the fact that the conformal time $\eta = -1/a(\eta)H$ runs from $-\infty$ (subhorizon) to -0 (superhorizon). The Penrose diagram is depicted in Fig. 1. Conformal invariance in \mathbb{R}^3 at $\eta = -\epsilon$ is connected to the isometry group $\text{SO}(1,4)$ of dS space. In this case, the dS isometry group acts as conformal group when fluctuations are superhorizon [3]. Hence, correlators are expected to be constrained by conformal invariance. Actually, a slice (\mathbb{R}^3) at $\eta = -\epsilon$ is employed to calculate the power spectrum in the superhorizon limit. On the other hand, one introduces the Bunch-Davies vacuum to compute the power spectrum in the subhorizon limit of $\eta \rightarrow -\infty$.

We wish to choose the Newtonian gauge of $B = E = 0$ and $\bar{E}_i = 0$ for cosmological perturbation around the dS background (9). In this case, the cosmologically perturbed metric can be simplified to be

$$ds^2 = a(\eta)^2 \left[- (1 + 2\Psi) d\eta^2 + 2\Psi_i d\eta dx^i + \left\{ (1 + 2\Phi) \delta_{ij} + h_{ij} \right\} dx^i dx^j \right] \quad (11)$$

with transverse-traceless tensor $\partial_i h^{ij} = h = 0$. Furthermore, two scalar perturbations are defined by

$$\phi_1 = 0 + \varphi_1, \quad \phi_2 = 0 + \varphi_2. \quad (12)$$

In order to obtain the perturbed Einstein equations, one can linearize the Einstein equation (2) directly around the dS spacetime as

$$\delta R_{\mu\nu}(h) - 3H^2 h_{\mu\nu} = 0 \rightarrow \bar{\nabla}^2 h_{ij} = 0, \quad (13)$$

which describes a massless gravitational wave propagation. Concerning two-metric scalars Ψ and Φ , their linearized Einstein equations imply that they are not physically propagating modes. In addition, we note that there is no coupling between $\{\Psi, \Phi\}$ and $\{\varphi_1, \varphi_2\}$ because

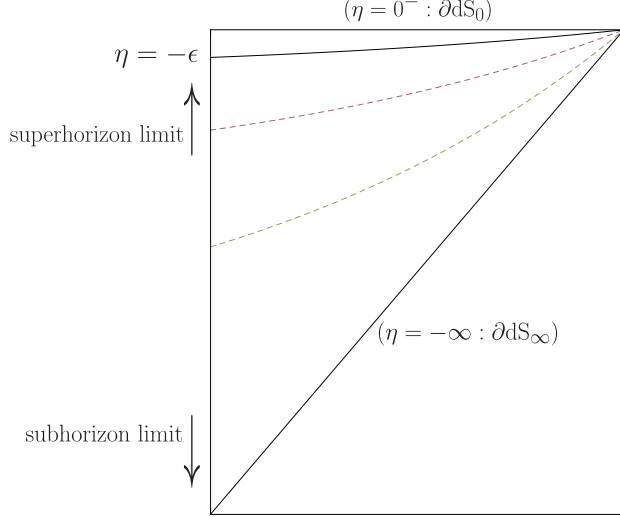


Figure 1: Penrose diagram of dS spacetime with the UV/IR boundaries ($\partial\text{dS}_{\infty/0}$) located at $\eta = -\infty$ and $\eta = -0$. A slice (\mathbb{R}^3) near $\eta = -\infty$ is introduced to compute the power spectrum in the subhorizon limit, while a slice (\mathbb{R}^3) at $\eta = -\epsilon$ is employed to calculate the power spectrum in the superhorizon limit.

of $\bar{\phi}_1 = \bar{\phi}_2 = 0$ in dS inflation. The vector Ψ_i is also a non-propagating mode since it has no kinetic term. The relevant linearized equations are those for two scalars

$$\bar{\nabla}^2 \varphi_1 = 0, \quad (14)$$

$$\bar{\nabla}^2 \varphi_2 = \mu \varphi_1, \quad (15)$$

which are combined to provide a degenerate fourth-order scalar equation

$$\bar{\nabla}^4 \varphi_2 = 0. \quad (16)$$

This is our main equation to be solved to obtain the power spectrum of a massless singleton during dS-inflation.

It seems appropriate to comment that Eqs.(14)-(16) are different from those of a massive singleton in [4]: $(\bar{\nabla}^2 - m^2)\varphi_1 = 0$, $(\bar{\nabla}^2 - m^2)\varphi_2 = \mu\varphi_1$, $(\bar{\nabla}^2 - m^2)^2\varphi_2 = 0$. We could not solve the massive singleton equation in the whole range of $\eta \in [-\infty, -0]$.

3 Propagation of massless singleton

In order to compute the complete power spectrum, we have to know the solution to singleton equations (15) and (16) in the whole range of $\eta \in [-\infty, -0]$. For this purpose, two scalars φ_i can be expanded in Fourier modes $\phi_{\mathbf{k}}^i(\eta)$

$$\varphi_i(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \phi_{\mathbf{k}}^i(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (17)$$

Eq.(15) leads to

$$\left[\frac{d^2}{d\eta^2} - \frac{2}{\eta} \frac{d}{d\eta} + k^2 \right] \phi_{\mathbf{k}}^1(\eta) = 0. \quad (18)$$

Introducing a new variable $z = -k\eta$, Eq.(18) can be rewritten as

$$\left[\frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + 1 \right] \phi_{\mathbf{k}}^1(z) = 0 \quad (19)$$

whose positive-frequency solution with the normalization $1/\sqrt{2k}$ is given by

$$\phi_{\mathbf{k}}^1(z) = \frac{H}{\sqrt{2k^3}} (i + z) e^{iz}. \quad (20)$$

This is the typical solution of a massless scalar propagating on dS spacetime.

On the other hand, plugging (17) into (16) leads to the fourth-order scalar equation

$$\left[\eta^2 \frac{d^2}{d\eta^2} - 2\eta \frac{d}{d\eta} + k^2 \eta^2 \right]^2 \phi_{\mathbf{k}}^2(\eta) = 0. \quad (21)$$

This equation can be expressed in terms of z as

$$\left[\frac{d^4}{dz^4} + 2 \left(1 - \frac{1}{z^2} \right) \frac{d^2}{dz^2} + \frac{4}{z^3} \frac{d}{dz} + \left(1 - \frac{2}{z^2} \right) \right] \phi_{\mathbf{k}}^2 = 0 \quad (22)$$

whose full solution is found to be

$$\phi_{\mathbf{k}}^2(z) = \left[\tilde{c}_2(i + z) + \tilde{c}_1 \left\{ 2i + (z - i) e^{-2iz} \text{Ei}(2iz) \right\} \right] e^{iz} \quad (23)$$

with two complex coefficients \tilde{c}_1 and \tilde{c}_2 . This is one of our main results which states that the solution (23) is an exact solution to the fourth-order equation (16). The c.c. of $\phi_{\mathbf{k}}^2$ is also a solution to (22). Here, $\text{Ei}(2iz)$ is the exponential-integral function of a purely imaginary number defined by [14]

$$\text{Ei}(2iz) = \text{Ci}(2z) + i\text{Si}(2z) - i\frac{\pi}{2}, \quad (24)$$

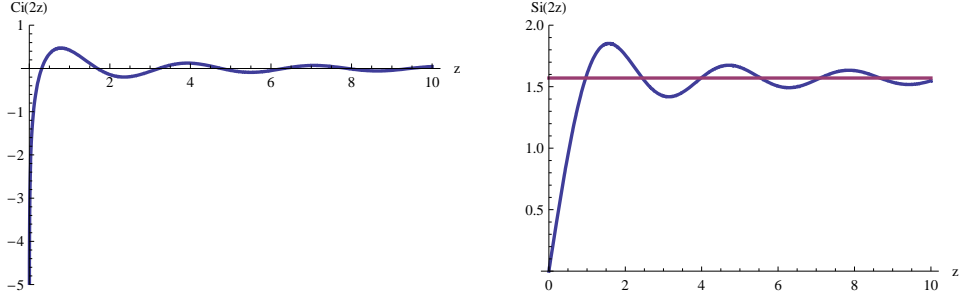


Figure 2: Cosine-integral and Sine-integral functions as functions of z . In the super horizon limit of $z \rightarrow 0$, one finds that $\text{Ci}[2z] \rightarrow \gamma + \ln[2z]$ and $\text{Si}[2z] \rightarrow 0$. On the other hand, one finds that $\text{Ci}[2z] \rightarrow \frac{\sin[2z]}{2z}$ and $\text{Si}[2z] \rightarrow \frac{\pi}{2} - \frac{\cos[2z]}{2z}$ in the subhorizon limit of $z \rightarrow \infty$.

where the cosine-integral and sine-integral functions are given by

$$\text{Ci}(2z) = - \int_{2z}^{\infty} \frac{\cos t}{t} dt \rightarrow \begin{cases} z \rightarrow 0 : \gamma + \ln[2z] + \sum_{k=1}^{\infty} \frac{(-1)^k (2z)^{2k}}{2k(2k)!} \\ z \rightarrow \infty : \frac{\sin(2z)}{2z} + \mathcal{O}\frac{1}{z^2} \end{cases}, \quad (25)$$

$$\text{Si}(2z) = \int_0^{2z} \frac{\sin t}{t} dt \rightarrow \begin{cases} z \rightarrow 0 : \sum_{k=1}^{\infty} \frac{(-1)^{k-1} (2z)^{2k-1}}{(2k-1)(2k-1)!} \\ z \rightarrow \infty : -\frac{\cos(2z)}{2z} + \frac{\pi}{2} + \mathcal{O}\frac{1}{z^2} \end{cases} \quad (26)$$

with the Euler's constant $\gamma = 0.577$. Their behaviors are depicted in Fig. 2. We note that $\text{Ei}(2iz)$ satisfies the fourth-order equation

$$\begin{aligned} (z-i)z^3 \frac{d^4 \text{Ei}(2iz)}{dz^4} - 4iz^4 \frac{d^3 \text{Ei}(2iz)}{dz^3} + 2z(i-z-4iz^2-2z^3) \frac{d^2 \text{Ei}(2iz)}{dz^2} \\ - 4(i-z-iz^2+2z^3) \frac{d \text{Ei}(2iz)}{dz} = 8e^{2iz} \end{aligned} \quad (27)$$

and its asymptotic behaviors are given by

$$\text{Ei}(2iz) \rightarrow \begin{cases} z \rightarrow 0 : \gamma + \ln[2z] - \frac{i\pi}{2} \\ z \rightarrow \infty : -\left[\frac{i}{2z} + \frac{1}{(2z)^2}\right] e^{2iz} \end{cases} \quad (28)$$

obtained from (24) together with (25) and (26).

It is worth to point out that the solution (23) is suitable for choosing the Bunch-Davies vacuum to give quantum fluctuations because it shows

$$\phi_{\mathbf{k}}^2(z) \rightarrow_{z \rightarrow \infty} \left[\left(\tilde{c}_2 + \frac{3}{2} \tilde{c}_1 \right) i + \tilde{c}_2 z \right] e^{iz}. \quad (29)$$

Then, Eq.(22) in the subhorizon limit of $z \rightarrow \infty$ reduces to a degenerate fourth-order equation which appeared in conformal gravity [15]

$$\left[\frac{d^2}{dz^2} + 1 \right]^2 \phi_{\mathbf{k},\infty}^2(z) = 0 \quad (30)$$

whose solution is given by

$$\phi_{\mathbf{k},\infty}^2(z) = (c'_1 + c'_2 z) e^{iz}. \quad (31)$$

We note that after redefining \tilde{c}_1 and \tilde{c}_2 , Eq.(29) leads to Eq.(31). The undetermined constants c'_1 and c'_2 shows a feature of solution to the fourth-order equation (30) when one compares these with the fixed solution (20) to the second order equation.

On the other hand, in the superhorizon limit of $z \rightarrow 0$, Eq.(22) reduces to

$$\left[\frac{d^4}{dz^4} - \frac{2}{z^2} \frac{d^2}{dz^2} + \frac{4}{z^3} \frac{d}{dz} \right] \phi_{\mathbf{k},0}^2 = 0, \quad (32)$$

whose solution is given by

$$\phi_{\mathbf{k},0}^2 = \bar{c}_1 + \bar{c}_2 \ln[2z] \quad (33)$$

with arbitrary constants \bar{c}_1 and \bar{c}_2 . Especially, the presence of $\ln[2z]$ dictates that (33) is the solution to the fourth-order equation (32). In deriving Eq.(32) from Eq.(22), we neglect the last term of $-\frac{2}{z^2}$ because it is subdominant in the limit of $z \rightarrow 0$. We note that the full solution (23) reduces to Eq.(33) in the limit of $z \rightarrow 0$:

$$\phi_{\mathbf{k},0}^2 = i \left[\tilde{c}_2 + (2 - \gamma + i\frac{\pi}{2}) \tilde{c}_1 \right] - i \tilde{c}_1 \ln[2z], \quad (34)$$

when choosing

$$\bar{c}_2 = -i \tilde{c}_1, \quad \bar{c}_1 = i \left[\tilde{c}_2 + \left(2 - \gamma + \frac{i\pi}{2} \right) \tilde{c}_1 \right]. \quad (35)$$

Finally, we may determine one coefficient \tilde{c}_1 by making use of Eq.(15) together with Eqs.(20) and (23):

$$\tilde{c}_1 = -\frac{\mu}{3H\sqrt{2k^3}}. \quad (36)$$

However, \tilde{c}_2 remains undetermined, but it will be determined by the Wronskian condition in the next section.

4 Power spectra

The power spectrum is the variance of singleton fluctuations due to quantum zero-point fluctuations. It is easily defined by the zero-point correlation function which could be computed when one chooses the Bunch-Davies vacuum state $|0\rangle$ in the subhorizon limit. The defining relation is given by

$$\langle 0 | \hat{\varphi}_a(\eta, 0) \hat{\varphi}_b(\eta, 0) | 0 \rangle = \int \frac{dk}{k} \mathcal{P}_{ab}, \quad (37)$$

where $k = \sqrt{\mathbf{k} \cdot \mathbf{k}}$ is the comoving wave number. Quantum fluctuations were created on all length scales with wave number k . Cosmologically relevant fluctuations start their lives inside the Hubble radius which defines the subhorizon: $k \gg aH$. On later, the comoving Hubble radius $1/(aH)$ shrinks during inflation while keeping the wavenumber k constant. Eventually, all fluctuations exit the comoving Hubble radius, they reside on the superhorizon region of $k \ll aH$ after horizon crossing.

For fluctuations of a massless scalar ($\bar{\nabla}^2 \delta\phi = 0$) and tensor ($\bar{\nabla}^2 h_{ij} = 0$) with different normalization originate on subhorizon scales and they propagate for a long time on superhorizon scales. This can be checked by computing their power spectra

$$\mathcal{P}_{\delta\phi} = \frac{H^2}{(2\pi)^2} [1 + z^2], \quad (38)$$

$$\mathcal{P}_h = 2 \times \left(\frac{2}{M_{\text{P}}} \right)^2 \times \mathcal{P}_{\phi} = \frac{2H^2}{\pi^2 M_{\text{P}}^2} [1 + z^2]. \quad (39)$$

To compute the singleton power spectrum, we have to know the commutation relations and the Wronskian condition. The canonical conjugate momenta are given by

$$\pi_1 = a^2 \varphi'_2, \quad \pi_2 = a^2 \varphi'_1. \quad (40)$$

The canonical quantization is accomplished by imposing equal-time commutation relations:

$$[\hat{\varphi}_1(\eta, \mathbf{x}), \hat{\pi}_1(\eta, \mathbf{y})] = i\delta^3(\mathbf{x} - \mathbf{y}), \quad [\hat{\varphi}_2(\eta, \mathbf{x}), \hat{\pi}_2(\eta, \mathbf{y})] = i\delta^3(\mathbf{x} - \mathbf{y}). \quad (41)$$

The two operators $\hat{\varphi}_1$ and $\hat{\varphi}_2$ are expanded in terms of Fourier modes as [6, 13, 15]

$$\hat{\varphi}_1(z, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} N \left[\left(i\hat{a}_1(\mathbf{k}) \phi_{\mathbf{k}}^1(z) e^{i\mathbf{k} \cdot \mathbf{x}} \right) + \text{h.c.} \right], \quad (42)$$

$$\hat{\varphi}_2(z, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \tilde{N} \left[\left(\hat{a}_2(\mathbf{k}) \phi_{\mathbf{k}}^1(z) + \hat{a}_1(\mathbf{k}) \phi_{\mathbf{k}}^2(z) \right) e^{i\mathbf{k} \cdot \mathbf{x}} + \text{h.c.} \right] \quad (43)$$

with N and \tilde{N} the normalization constants. Plugging (42) and (43) into (41) determines the relation of normalization constants as $N\tilde{N} = 1/2k$ and commutation relations between $\hat{a}_a(\mathbf{k})$ and $\hat{a}_b^\dagger(\mathbf{k}')$ as

$$[\hat{a}_a(\mathbf{k}), \hat{a}_b^\dagger(\mathbf{k}')] = 2k \begin{pmatrix} 0 & -i \\ i & 1 \end{pmatrix} \delta^3(\mathbf{k} - \mathbf{k}'), \quad (44)$$

where we observe a Jordan cell structure. This is the typical commutation relations appeared when one quantizes a degenerate Pais-Uhlenbeck fourth-order oscillator [6]. Here the commutation relation of $[\hat{a}_2(\mathbf{k}), \hat{a}_2^\dagger(\mathbf{k}')] is implemented by the Wronskian condition. The Wronskian condition for $\phi_{\mathbf{k}}^1(z)$ and $\phi_{\mathbf{k}}^2(z)$ leads to$

$$\begin{aligned} & a^2 \left(\phi_{\mathbf{k}}^1 \frac{d\phi_{\mathbf{k}}^{2*}}{dz} - \phi_{\mathbf{k}}^{2*} \frac{d\phi_{\mathbf{k}}^1}{dz} + \phi_{\mathbf{k}}^{1*} \frac{d\phi_{\mathbf{k}}^2}{dz} - \phi_{\mathbf{k}}^2 \frac{d\phi_{\mathbf{k}}^{1*}}{dz} \right) \\ &= \sqrt{\frac{k}{2}} \frac{1}{H} \left[2i(\tilde{c}_2 - \tilde{c}_2^*) - (\tilde{c}_1 + \tilde{c}_1^*) \left(\frac{1}{z^3} + \frac{3}{z} \right) \right] = \frac{1}{k}. \end{aligned} \quad (45)$$

To satisfy the above relation, let us impose

$$\tilde{c}_1 = -\tilde{c}_1^*, \quad \tilde{c}_2 = -\frac{iH}{2\sqrt{2k^3}}. \quad (46)$$

At this stage, it is worth to note that the Wronskian normalization condition was originally designed for the second-order theory. In the subhorizon limit of $z \rightarrow \infty$, the fourth-order contribution is nothing, while it blows up unless \tilde{c}_1 is purely imaginary in the superhorizon limit of $z \rightarrow 0$. Hence, we may neglect the fourth-order contribution to the Wronskian condition by choosing \tilde{c}_1 to be purely imaginary. Considering (36), one may determine

$$\tilde{c}_1 = -i \frac{2H}{3\sqrt{2k^3}} \quad (47)$$

by choosing $\mu = 2iH^2$. We note here that choosing $\tilde{c}_1 = i \frac{2H}{3\sqrt{2k^3}}$ leads to the positive power spectrum ($\mathcal{P}_{22} > 0$) in the whole range z , which contradicts to the negative power spectrum of a fourth-order scalar theory.

Then, we could easily find that

$$\mathcal{P}_{11} = 0, \quad \mathcal{P}_{12}(z) = \mathcal{P}_{21}(z) = \frac{k^3}{2\pi^2} |\phi_{\mathbf{k}}^1|^2 = \frac{H^2}{(2\pi)^2} [1 + z^2]. \quad (48)$$

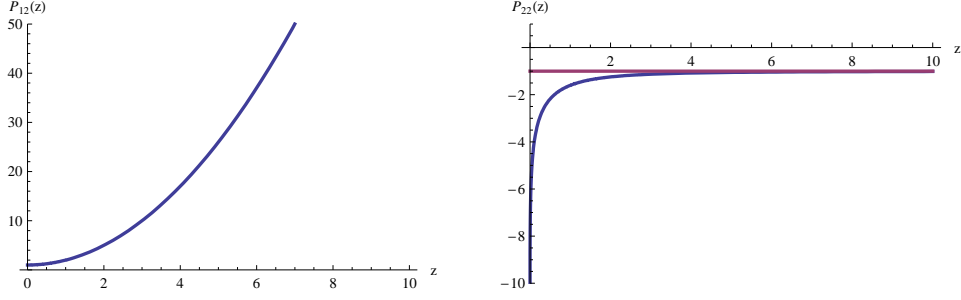


Figure 3: Power spectra \mathcal{P}_{12} and \mathcal{P}_{22} as functions of z for $H^2 = (2\pi)^2$. In the superhorizon limit of $z \rightarrow 0$, one finds that $\mathcal{P}_{12} \rightarrow 1$ while $\mathcal{P}_{22} \rightarrow -\infty$. On the other hand, $\mathcal{P}_{12} \rightarrow \infty$ and $\mathcal{P}_{22} \rightarrow -1$ in the subhorizon limit of $z \rightarrow \infty$.

However, the power spectrum \mathcal{P}_{22} takes a complicated form

$$\begin{aligned}
\mathcal{P}_{22}(z) &\equiv \mathcal{P}_{22}^{(1)}(z) + \mathcal{P}_{22}^{(2)}(z) \\
&= \frac{k^3}{2\pi^2} \left[|\phi_{\mathbf{k}}^1|^2 + i(\phi_{\mathbf{k}}^1 \phi_{\mathbf{k}}^{2*} - \phi_{\mathbf{k}}^2 \phi_{\mathbf{k}}^{1*}) \right] \\
&= \left(\frac{H}{2\pi} \right)^2 \left[1 + z^2 - \left\{ 1 + z^2 + 4i\tilde{c}_1 \frac{\sqrt{2k^3}}{H} + 2i\tilde{c}_1 \frac{\sqrt{2k^3}}{H} \text{Re}[f(z)] \right\} \right] \\
&= -\frac{4}{3} \left(\frac{H}{2\pi} \right)^2 \left[2 + \text{Re}[f(z)] \right],
\end{aligned} \tag{49}$$

where $f(z)$ is given by

$$f(z) = e^{2iz}(i+z)^2 \text{Ei}(-2iz). \tag{50}$$

This is another of our main results: power spectrum of massless singleton is explicitly expressed in terms of the exponential-integral function. Fig. 3 indicates the behaviors of $\mathcal{P}_{12}(z)$ and $\mathcal{P}_{22}(z)$ generated during dS inflation. We note that the former shows a typical power spectrum for a massless scalar ($\delta\phi, \varphi_1$) or graviton (h), while the latter indicates a power spectrum of the singleton (φ_2). It is reasonable to assist that the power spectrum of \mathcal{P}_{22} is negative because it corresponds to that of a purely fourth-order scalar theory. That is, one could not avoid to find ghost-instability when computing the power spectrum of a fourth-order derivative scalar theory during dS inflation.

In the subhorizon limit of $z \rightarrow \infty$, one finds a negatively scale-invariant spectrum

$$\mathcal{P}_{22}^{z \rightarrow \infty} = - \left(\frac{H}{2\pi} \right)^2 \tag{51}$$

because $\mathbf{Re}[f(z)] \rightarrow -\frac{5}{4}$ in this limit. We note that the power spectrum of the scale-invariant scalar tensor theory is given by [16]

$$\mathcal{P}_{\text{SIST}} = \frac{1}{2(2\pi)^2}. \quad (52)$$

Taking $f(z)$ in the superhorizon limit of $z \rightarrow 0$

$$f(z) \rightarrow_{z \rightarrow 0} \left[-\gamma - \ln[2z] + (1 - \gamma)z^2 \right] + i \left[-\frac{\pi}{2} + 2z - \frac{\pi z^2}{2} \right], \quad (53)$$

the power spectrum (49) of massless singleton leads to

$$\begin{aligned} \mathcal{P}_{22}^{z \rightarrow 0}(z) &= -\frac{4}{3} \left(\frac{H}{2\pi} \right)^2 \left(2 - \gamma - \ln[2z] \right) \\ &= \frac{4}{3} \left(\frac{H}{2\pi} \right)^2 \left(\ln[z] - 0.73 \right), \end{aligned} \quad (54)$$

which explains why $\mathcal{P}_{22}(z)$ blows up negatively as $z \rightarrow 0$ in Fig. 3. On the other hand, one has a power spectrum for a massless scalar

$$\mathcal{P}_{12}^{z \rightarrow 0} = \left(\frac{H}{2\pi} \right)^2. \quad (55)$$

5 Discussions

We have obtained the exact solution and computed the complete power spectrum (49) of a singleton expressed in term of the exponential-integral function by solving the degenerate fourth-order equation and by requiring the Pais-Uhlenbeck quantization scheme for a degenerate fourth-order oscillator.

Its two asymptotic behaviors are quite different from those [(52) and (55)] of a massless scalar. In the subhorizon limit $z \rightarrow \infty$, the power spectrum (51) of a singleton is a negatively scale-invariant one which is opposite to (52) of scale-invariant scalar-tensor theory [16], while it blows up (negatively divergent) in the superhorizon limit of $z \rightarrow 0$ as is shown in (54). This indicates a feature of purely fourth-order derivative scalar theory in dS spacetime [3].

Even though our computation was based on the dS inflation, the above asymptotic features have suggested that the Einstein-singleton theory including a fourth-order scalar theory is not a good candidate for a slow-roll (quasi-dS) inflation model.

Finally, we discuss some issues relevant to our model.

- Ghost-instability of the model

Since S_S in (1) reduces to the fourth-order derivative scalar theory (6), we worry about the ghost-instability problem. Using the Pais-Uhlenbeck quantization scheme for a degenerate fourth-order oscillator in dS spacetime, we have found the negative power spectrum $\mathcal{P}_{22}(z)$ in (49), depicted in Fig 3. In the subhorizon limit of $z \rightarrow \infty$, we have obtained a negatively scale-invariant power spectrum (51) which indicates the ghost instability clearly. On the other hand, $\mathcal{P}_{22}(z)$ blows negatively up in the superhorizon limit of $z \rightarrow 0$. This indicates that the singleton theory is a fourth-order derivative scalar theory which must contain a ghost state.

- Problem of exit mechanism

The dS inflation is driven by the cosmological constant Λ which is a non-dynamical quantity. Hence, this corresponds to an eternal inflation and thus, there is no natural way to exit the inflationary phase. This is a handicap of dS inflation. In the slow-roll inflation (quasi-dS inflation), however, the inflaton plays an essential role in exiting the inflationary phase.

- Is $\mu = 2iH^2$ a mass square of ϕ_1 ?

In order to obtain Eq. (47), we specified $\mu = 2iH^2$. Recalling the definition of μ in (1), it seems that μ plays the role of the mass square of ϕ_1 . However, this is not true. μ is just a parameter of connecting ϕ_2 with ϕ_1 to get the fourth-order derivative equation for ϕ_2 from a mixed kinetic term. If one wishes to have a massive singleton, one has to include a potential term of $m^2\phi_1\phi_2$ [4]: $(\bar{\nabla}^2 - m^2)\varphi_1 = 0$, $(\bar{\nabla}^2 - m^2)\varphi_2 = \mu\varphi_1$, $(\bar{\nabla}^2 - m^2)^2\varphi_2 = 0$.

- Slow-roll inflation in the Einstein-singleton theory

If one wishes to consider the slow-roll inflation in the Einstein-singleton theory S_{ES} including the potential of $m^2\phi_1\phi_2$, the Einstein equation takes the form of $G_{\mu\nu} = T_{\mu\nu}^m/M_P^2$ which provides the energy density $\rho = \dot{\phi}_1\dot{\phi}_2 + (m^2\phi_1\phi_2 + \mu\phi_1^2/2)$ and the pressure $p = \dot{\phi}_1\dot{\phi}_2 - (m^2\phi_1\phi_2 + \mu\phi_1^2/2)$. The first and second Friedmann equations are given by $H^2 = \frac{\rho}{3M_P^2}$ and $\dot{H} = -\frac{\rho+p}{2M_P^2}$. Even though this model is similar to two-field inflation model with the chaotic potentials, this is not the case because their full scalar equations are given by $\ddot{\phi}_1 + 3H(t)\dot{\phi}_1 + m^2\phi_1 = 0$ and $\ddot{\phi}_2 + 3H(t)\dot{\phi}_2 + m^2\phi_2 = -\mu\phi_1$ which are combined to give a fourth-order equation of $(\frac{d^2}{dt^2} + 3H(t)\frac{d}{dt} + m^2)^2\phi_2 = 0$. It conjectures that their slow-roll equations are quite different from those of two-field inflation. Furthermore, it requires a non-trivial task to perform the cosmological perturbations around the slow-roll inflation

instead of the dS inflation. Especially, it is important to define the curvature perturbation \mathcal{R} in the Einstein-singleton theory. It was given by $\mathcal{R} = -H\delta\phi/\dot{\phi}$ for the single-field inflation in spatially flat gauge, while it takes the form of $\mathcal{R}_S = -H[\varphi_1/\dot{\phi}_1 + \varphi_2/\dot{\phi}_2]$ for the singleton inflation. For example, the power spectrum appeared in dS spacetime with $\dot{\phi}_1 = \dot{\phi}_2 = 0$ [3, 4] was given by $\mathcal{P}_{\varphi_2\varphi_2}^m \sim z^{2w}(1 + 2\ln[z])$ with $w = 3/2 - \sqrt{9/4 - m^2/H^2}$ in the superhorizon limit. However, we remain “cosmological perturbations of the Einstein-singleton theory around the slow-roll inflation” as a future work, worrying about the appearance of the ghost states. This is so because the strange asymptotic behavior of power spectrum of $\mathcal{P}_{\varphi_2\varphi_2}$ indicates a negatively divergent behavior in the superhorizon limit of $z \rightarrow 0$, which reflects that the Einstein-singleton theory includes a fourth-order derivative scalar theory. Furthermore, there is no way to avoid a ghost-instability in the whole range of z . Thus, our result during dS inflation suggests that the Einstein-singleton theory is not considered as a model for developing a slow-roll inflation because a negative power spectrum of curvature perturbation ($\mathcal{P}_{\mathcal{R}_S\mathcal{R}_S} < 0$) persists in the slow-roll inflation. This is because φ_2 satisfies a fourth-order differential equation during the slow-roll inflation.

References

- [1] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1502.02114 [astro-ph.CO].
- [2] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980). doi:10.1016/0370-2693(80)90670-X
- [3] A. Kehagias and A. Riotto, Nucl. Phys. B **864**, 492 (2012) doi:10.1016/j.nuclphysb.2012.07.004 [arXiv:1205.1523 [hep-th]].
- [4] Y. S. Myung and T. Moon, JHEP **1410**, 137 (2014) doi:10.1007/JHEP10(2014)137 [arXiv:1407.7742 [gr-qc]].
- [5] A. Pais and G. E. Uhlenbeck, Phys. Rev. **79**, 145 (1950).
- [6] P. D. Mannheim and A. Davidson, Phys. Rev. A **71**, 042110 (2005) [hep-th/0408104].
- [7] Y. W. Kim, Y. S. Myung and Y. J. Park, Phys. Rev. D **88**, 085032 (2013) doi:10.1103/PhysRevD.88.085032 [arXiv:1307.6932].
- [8] M. Flato and C. Fronsdal, Commun. Math. Phys. **108**, 469 (1987).
- [9] A. M. Ghezelbash, M. Khorrami and A. Aghamohammadi, Int. J. Mod. Phys. A **14**, 2581 (1999) [hep-th/9807034].
- [10] I. I. Kogan, Phys. Lett. B **458**, 66 (1999) [hep-th/9903162].
- [11] Y. S. Myung and H. W. Lee, JHEP **9910**, 009 (1999) [hep-th/9904056].
- [12] D. Grumiller, W. Riedler, J. Rosseel and T. Zojer, J. Phys. A **46**, 494002 (2013) [arXiv:1302.0280 [hep-th]].
- [13] V. O. Rivelles, Phys. Lett. B **577**, 137 (2003) doi:10.1016/j.physletb.2003.10.039 [hep-th/0304073].
- [14] M. Abramowitz and A. Stegun, Handbook of Mathematical functions, (Dover publications, New York, 1970).
- [15] Y. S. Myung and T. Moon, Mod. Phys. Lett. A **30**, no. 32, 1550172 (2015) doi:10.1142/S0217732315501722 [arXiv:1501.01749 [gr-qc]].

- [16] Y. S. Myung and Y. J. Park, Eur. Phys. J. C **76**, no. 2, 79 (2016)
doi:10.1140/epjc/s10052-016-3924-0 [arXiv:1508.04188 [gr-qc]].